A Parton Shower Model for
Hadronic Two-Photon Process in e^+e^- Scatterings

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ABSTRACT

A new model of QCD parton shower is proposed which is dedicated to two-photon process in e^+e^- scattering. When hadron jets are produced, the photon may resolve into quark-antiquark pairs so that the structure functions of the photon should be introduced. Based on the Altarelli-Parisi equation for these functions, an algorithm is formulated that allows us to construct a model for parton showers for the photon. Our model consists of two parts, one of which describes the deep inelastic scattering of the photon and the other one the scattering of two quasi-real photons. Using the model some results are presented on parton distributions and jet production.

Section 1. Introduction

Recent extensive studies in TRISTAN experiments on the two-photon process in e^+e^- scattering, particularly on jet production[1, 2], have stimulated more detailed theoretical investigations in QCD[3, 4, 5]. However these calculations have been limited to the inclusive case so far. In order to make a precise comparison between experiments and theory, one needs an event generator where one starts with the generation of partons and then convert them into hadrons and other stable particles. Distributions of generated partons are completely determined by perturbative QCD. To generate partons one has two possibilities. The first one is the matrix element method which uses the fixed order perturbative calculation and the other one is the parton shower method[6] based on the Altarelli-Parisi(AP) equation. In the latter one can take into account all the leading contributions from the collinear singularities without limitation in the order of the coupling.

The parton shower model has been demonstrated to be powerful and indispensable for quantitative studies in e^+e^- annihilation and other processes. It is also true for the two-photon experiments and it is highly desirable to develop an event generator that includes parton showers, since recent experiments provide data with rather good accuracy. In this paper we construct such a parton shower model dedicated to the two-photon process.

One of the main features of this parton shower is the resolution of the photon, i.e. the case where the photon splits into a quark-antiquark pair at the beginning of the reaction. Partons inside the photon have two sources. One is taken into account by the so called vector meson dominance model(VDM) and the other one is the perturbative photon. They correspond to the solutions of the homogeneous and inhomogeneous AP equations, respectively. In terms of parton language, the VDM contribution arises when the photon converts into the vector mesons like ρ^0 and ω and then the partons inside these mesons undergo QCD evolution. On the other hand the perturbative photons are those that decay into quark-antiquark pairs and then these quarks evolve according to perturbative QCD. We will call them the

VDM and the perturbative photon parts in the following.

In the case of two quasi-real photon scattering we have another contribution where the original photon undergoes a hard scattering. This shall be called the direct part.

The present model is limited to the leading-logarithmic(LL) approximation. An extension to the next-to-leading logarithmic case is, however, not difficult[7]. One reason for this limitation is that experimental data are not yet precise enough to determine the QCD scale parameter Λ and another reason is that the most important contributions of the higher order are already included into the model as discussed in[7].

To apply the model to actual reactions one has to distinguish two situations(see Fig.1). The first one is the deep inelastic scattering of a photon, i.e. one photon is quasi-real while the other has a large virtual mass squared $-Q^2$. One requirement from perturbative QCD is that this virtual mass squared is much larger than the QCD scale parameter and hadron mass scale. Hence one has to assume $Q^2 \geq 5 - 10$ GeV^2. The second case is for two quasi-real photon scattering. In this process one can get reliable predictions from QCD calculations only when jets with large p_T^2 are produced. Notice that our event generator produces only partons, not hadrons. Hence the VDM contributions express only the part related to the hard scattering. We do not consider the low p_T scattering in the hadron interaction.

In the next section we show how to formulate the parton shower model, starting from the AP equations. In section 3 a detailed description of the model will be presented. There we discuss the differences between the models of deep inelastic scattering and two quasi-real photon scattering. These are related not to the algorithm of the model but the way to apply it. Section 4 shows some results obtained by Monte Carlo simulations of the parton shower. One will see that it reproduces the same distributions as the conventional QCD calculations for inclusive observables. The last section is devoted to summary and discussion.

Section 2. Basic formulation

The algorithm of the parton shower is based on the AP equation, which governs the evolution of the photon structure function. First we solve this equation in the way most convenient to our purpose, which gives us a clear basis for the construction of the shower model. For simplicity we limit the discussion here to the non-singlet case within the LL approximation.

The AP equation for $q_{NS}(x, Q^2)$, the non-singlet quark distribution in the photon[3, 4], is given by

$$\frac{dq_{NS}(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}^{(0)}(x/y) q_{NS}(y, Q^2) + \frac{\alpha}{2\pi} k_{NS}^{(0)}(x), \qquad (1)$$

Here $P_{qq}^{(0)}(x)$ is the quark splitting function. This equation differs from the usual one by the existence of an inhomogeneous term, $k_{NS}^{(0)}(x)$. It is known that this term has a x-dependence of the form $x^2 + (1 - x)^2$. The QED coupling is denoted as α and that of QCD is given by

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln Q^2/\Lambda^2} \equiv \frac{\alpha_0}{\ln Q^2/\Lambda^2}, \qquad \beta_0 = 11 - \frac{2}{3} N_F. \qquad (2)$$

where N_F is the number of flavors. Eq.(1) can be solved easily if one takes its moments:

$$q_{NS}(n, Q^2) = \int_0^1 dx \, x^{n-1} q_{NS}(x, Q^2), \qquad (3)$$

$$
\begin{aligned}
q_{NS}(n, Q^2) &= q_{NS}(n, Q_0^2) \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{\alpha_0 d(n)} \\
&+ \frac{\alpha}{\alpha_s(Q^2)} \frac{a_{NS}(n)}{\alpha_0 d(n) - 1} \left[\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{\alpha_0 d(n) - 1} - 1 \right],
\end{aligned} \qquad (4)
$$

where $d(n)$ is the moment of $P_{qq}^{(0)}(x)/2\pi$ and $a_{NS}(n)$ is that of $(\alpha/2\pi) k_{NS}^{(0)}(x)$.

In order to show the dependence on n in a more transparent form, we introduce an integral to rewrite the second term,

$$q_{NS}(n, Q^2) = q_{NS}(n, Q_0^2) e^{\alpha_0 d(n) \bar{s}} + \frac{\alpha}{\alpha_s(Q^2)} a_{NS}(n) \int_0^{\bar{s}} d\eta \, e^{-\eta} e^{\alpha_0 d(n) \eta}, \qquad (5)$$

where

$$\overline{s} = \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} = \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}. \tag{6}$$

This is the final form of the solution in the moment space.

To get the corresponding solution in the x-space, we make the inverse Mellin transformation defined by the formula

$$f(x) = \int_{r_0-i\infty}^{r_0+i\infty} \frac{dn}{2\pi i} x^{-n} f(n), \tag{7}$$

where r_0 is a real number which defines the location of the integration path. The result can be cast into the following form

$$
\begin{aligned}
q_{NS}(x, Q^2) &= \int_x^1 \frac{dy}{y} K_{NS}^{(0)}(x/y, \overline{s}) q_{NS}(y, Q_0^2) \\
&+ \ln(Q^2/\Lambda^2) \int_0^{\overline{s}} d\eta e^{-\eta} \int_x^1 \frac{dy}{y} K_{NS}^{(0)}(x/y, \eta) \frac{\alpha}{2\pi} k_{NS}^{(0)}(y).
\end{aligned} \tag{8}
$$

Here we have introduced the QCD kernel function $K_{NS}^{(0)}(x, \overline{s})$ defined in [8].

$$K_{NS}^{(0)}(x/y, \overline{s}) = \int_{r_0-i\infty}^{r_0+i\infty} \frac{dn}{2\pi i} x^{-n} e^{\alpha_0 d(n)\overline{s}}. \tag{9}$$

This kernel function is parametrized in a simple form so that it is easy to carry out convolution integrals in Eq.(8) numerically. The physics meaning of each term in this solution is clear. The first term expresses the contribution from VDM, while the second contains the effect of the perturbative photon.

In order to translate the Eq.(8) into the language of parton showers, it is convenient to change the integral variable η as follows,

$$\eta = \ln \frac{\ln Q^2/\Lambda^2}{\ln K^2/\Lambda^2}, \tag{10}$$

where $-K^2$ can be regarded as the squared virtual mass of the quark that is emitted from the photon. By this change of variable, one can rewrite the solution in the following way:

$$
\begin{aligned}
q_{NS}(x, Q^2) &= \int_x^1 \frac{dy}{y} K_{NS}^{(0)}(x/y, \overline{s}) q_{NS}(y, Q_0^2) \\
&+ \int_{Q_0^2}^{Q^2} \frac{dK^2}{K^2} \int_x^1 \frac{dy}{y} K_{NS}^{(0)}(x/y, \eta(K^2)) \frac{\alpha}{2\pi} k_{NS}^{(0)}(y).
\end{aligned} \tag{11}
$$

The second term of this equation says that at the beginning the photon resolves into a quark-antiquark pair and then the spacelike quark(antiquark) evolves according to the usual QCD branching, because $K_{NS}^{(0)}(x/y, \eta(K^2))$ expresses the evolution from K^2 to Q^2. Based on this observation we can formulate an algorithm for the QCD evolution of the photon. So far the discussion is limited to the non-singlet case, but the extension to the singlet case can be done in a parallel way and our model is, needless to say, constructed to contain both contributions.

Section 3. The model

In this section we describe the model in some details. We assume the forward evolution method for the spacelike QCD evolution, as there is no essential difference between the forward evolution and the backward one[9]. One reason is, however, that the former makes it easier to write programs according to Eq.(1).

The photon branching differs slightly from the other processes such as e^+e^- annihilation or deep inelastic ep scattering. In the latter the total energy is conserved and the anomalous dimension corresponding to it vanishes identically. In the shower model which has been developed by one of the authors, the algorithm crucially relies on this conservation[10]. In other words the evolution is considered *not* for the particle distributions $q(x, Q^2)$, *but* for the energy distributions $xq(x, Q^2)$. In the present case, however, Eq.(1) is inhomogeneous and the second moment of $k_{NS}^{(0)}(y)$ does not vanish so that the energy sum rule is broken. Hence in the photon shower model, one has to abandon parton energy conservation. Of course the total energy including that of the photon should be conserved.

$$\int_0^1 dx x[\sum_f (q_f(x, Q^2) + \overline{q}_f(x, Q^2)) + G(x, Q^2)]$$

$$= \int_{Q_0^2}^{Q^2} \frac{dK^2}{K^2} \frac{\alpha}{2\pi} \int_0^1 dx x k_{NS}^{(0)}(x) \tag{12}$$

Therefore the quantity appearing on the right-hand side of this equation is taken into account as *the weight of an event*.

Let us state the algorithm step by step. It can be divided into the spacelike evolution and the hard scatterings that take place afterwards. Procedures of the spacelike evolution can be summarized as follows;

(A) Choose Q^2.

(B) Calculate the energy in the VDM, which is independent of Q^2, and the energy in the perturbative photon by using Eq.(12). The sum of these energies is kept as the weight of an event.

(C) Determine the process, either the VDM or the perturbative photon, according to the ratio of energies.

(D) If the VDM is chosen, make the usual QCD evolution up to Q^2 from Q_0^2, the cut-off momentum.

(E) In the case of the perturbative photon, determine the virtual mass squared K^2 according to the probability dK^2/K^2. Then choose a flavor of quark according to the ratio of charges squared.

(F) For the selected flavor of quark or antiquark, make the usual QCD evolution up to Q^2 from K^2.

The same algorithm is used for the deep inelastic photon scattering and two quasi-real photon scattering with large p_T^2. The hard scattering parts, however, completely differ from each other.

First we discuss the deep inelastic case. Here the QCD evolution gives the structure function $F_2(x, Q^2)$ of the photon, which is connected to the cross section in e-γ scattering through the following relation[4],

$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}(1-y)F_2(x, Q^2),$$

$$y = Q^2/(xs_\gamma),$$

where s_γ is the energy squared of the e-γ system.

In the hard scattering of this process the gluon does not participate but the quarks do with the weight of the squared charge. Determination of the four momenta is

completed after the timelike evolution is developed and the energy of the initial photon is fixed by the spectrum function of the photon in the electron, which is given by the Weizsäcker-Williams approximation. One of its simplest form is given by

$$F_{\gamma/e}(z, E) = \frac{\alpha}{\pi z}(1 + (1 - z)^2)(\ln \frac{E}{m_e} - \frac{1}{2}).$$

In the analysis of actual data more sophisticated versions are used.

Next we discuss the scattering of two quasi-real photons. In this case contributions from the direct photon should be taken into account. The magnitude of the cross sections for various hard processes is necessary to determine the weight of events. They are different depending on the type of partons involved in the hard scattering. For example, if the initial state of the hard scattering is a gluon and a direct photon, the final state is a quark and an antiquark. Then we have to know the cross section.

$$\frac{d\sigma(\hat{s})}{dp_T^2}(G + \gamma \to q + \overline{q}). \tag{13}$$

Here $\hat{s}$ is the energy squared of this hard scattering and the hard scale Q^2 can be identified as the transverse momentum squared of the scattered partons, $Q^2 = p_T^2$.

As the possible initial states of the hard scatterings we have to consider q-q, q-$\overline{q}$, q-G, G-G, q-γ, G-γ and γ-γ.

In order to generate events in the most effective way with $O(1)$ weights, we adopt the following method. The cross sections for hard scattering with photons of direct, direct-resolved, doubly-resolved are in the order of α^2, $\alpha\alpha_s$ and α_s^2, respectively. Consequently the corresponding events must have quite different weights. In the total cross section, however, the order of α is common for these three channels. In other words the probabilities that a single real photon resolves or not is $O(\alpha)$: $O(1)$. Hence we can rearrange α by removing it from the hard cross section to the probability of the photon branching. Thus we have the ratio $O(\alpha) : O(\alpha)$ to branch or not for a photon and $O(1) : O(\alpha_s) : O(\alpha_s^2)$ for the hard cross sections, which are comparable in magnitude. This rearrangement then allows us to generate events

with good efficiency. We use $d\sigma^0/dp_T^2 = \pi/p_T^4$ as the reference cross section. If any hard cross section is equal to this, the event is generated with weight $= 1$.

We like to point out also that the argument of α_s in the hard cross section is p_T^2, not $\hat{s}$ in our model(in contrast to the case of QCD branching where the virtuality K^2 enters into the argument). This is because there appear some t-channel diagrams in the hard scattering, and when p_T^2 is smaller than $\hat{s}$, the cross section is dominated by the t-channel contribution. The running effect comes from the virtual corrections in the t-channel propagator and its scale is developed to p_T^2.

When the final states of the hard scattering is fixed, we make timelike evolutions from Q^2 for timelike partons. Then we fix the energy of the initial photons in the electron and the positron, and thus we can determine the four-momenta of all partons.

Finally a comment is given about the higher order contributions. In the literature it has been emphasized that they are important for quantitative analysis[11, 12]. It has been known, however, from the study of e^+e^- annihilation, that the most important contributions in the next-to-leading logarithmic terms can be included in the LL shower by using the transverse momentum squared p_T^2 as the argument of the QCD coupling α_s and imposing the angular ordering[7]. This is also valid for the photon evolution we are considering. These effects are easily implemented into our model.

Section 4. Results

We present results on parton distributions. These distributions are usually obtained by integrating the AP equation numerically. The parton shower provides us an another method to get them. Though this equivalence has been well established, it will be instructive to understand the algorithm, by comparing the results of the parton shower with those obtained by the conventional method. Many kinds of distributions[13] have been proposed in the literature. First we use those given in

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ref. [14] for the LL approximation, because their distributions are parametrized in simple functions. In order to make precise comparison, we calculate parton distributions by the shower model with a fixed cutoff ϵ to regularize the infrared singularity. Results are shown in Fig.2, where one can see good agreement which shows that the shower algorithm yields the same distribution as obtained by usual methods to solve the AP equation directly.

Next we discuss the higher order effects. As mentioned in section 3, the most important effect beyond the leading order can be easily included into the LL model by using the transverse momentum squared p_T^2 as the argument of the running coupling. To show it, we calculate the structure function, $F_2(x, Q^2)$ by the parton shower with the transverse momentum p_T^2 and the virtual mass K^2 as the arguments. As the full next-to-leading logarithm(NLL) solution we take the distributions given in [12]. Here one should note that this comparison is not trivial. One reason is that the parton distributions in the NLL order are scheme-dependent and the definition of the initial distributions is not unique. Another reason is that in ref.[12] the evolution started at too small Q_0^2 so that contributions beyond the NLL order cannot be ignored. Therefore for the comparison we start the shower evolution at $Q^2 = 4\mathrm{GeV}^2$ using $F_2(x, 4\mathrm{GeV}^2)$ as the initial parton distribution. From Fig.3 we find that our shower model with the running coupling of p_T^2 can reproduce the NLL results qualitatively. This agreement is more remarkable for higher Q^2. However considering the fact that we are able to get a qualitative agreement between the NLL calculation and the LL shower together with the fact that there still remains non negligible experimenatal errors in the present data, there seems no need for the NLL shower model for the moment, though it is in principle possible[7].

Next we will discuss jet production in two quasi-real photon processes in e^+e^- scattering. In order to check the program, we compare the results of our model with the inclusive QCD calculations obtained by integrating the parton distributions $f_i(x, p_T^2)$ with the hard cross sections σ_{ij}^H, where i, j denote photon, quarks and

gluon.

$$\frac{d\sigma}{dp_T^2} = \int_0^1 dz_1 F_{\gamma/e}(z_1, E) \int_0^1 dz_2 F_{\gamma/e}(z_2, E)$$

$$\times \sum_{i,j} \int_0^1 dx_1 f_i(x_1, p_T^2) \int_0^1 dx_2 f_j(x_2, p_T^2) \delta(z_1 z_2 x_1 x_2 s - \hat{s}) \frac{d\sigma_{ij}^H(\hat{s})}{dp_T^2} \quad (14)$$

It should be noted that it is *not* trivial for our model to get the cross section, because the parton distributions are obtained as a result of the generated events[15], not being assumed at the beginning of the calculation. Figure 4, where the integrated cross sections, $\int_{p_{T\ min}^2}^{s/4} dp_T^2 d\sigma/dp_T^2$ are given, shows consistency of our model. Here to make a detailed comparison, we present the results for hard scattering of quark-photon and gluon-photon. The contributions from other processes are small(parton-parton scattering) in TRISTAN energy range, or are trivial(photon-photon scattering).

The parton shower model has an advantage, when one wants to make topological analyses of the final particles. As a typical example of these kinds of analyses, we take the thrust distribution. Since the system of final partons is boosted along z-direction, we analyze events using transverse momentum. Similarly to the case in e^+e^- annihilation, the thrust distribution changes as the energy increases(see Fig.5). Note again that in the present analysis we make no hadronization and the results are not fully meaningful as a physical prediction. The hadronization should be included for quantitative analysis of actual data.

Section 5. Summary and discussions

In this paper we proposed an algorithm of parton shower model for two-photon processes in e^+e^- scattering and constructed a Monte Carlo model according to it. We formulated the algorithm using AP equations and the kernel function method[8] for the QCD evolution. In the construction of the model, we adopted the forward spacelike evolution for the energy distributions of partons $xq(x, Q^2)$ by taking account of the breaking of energy conservation due to the inhomogeneous term in the

AP equations for the photon.

Using this Monte Carlo model, we have shown the parton distributions of the photon and some results on jet production in the two-photon process. These results clearly demonstrated that the model is consistent with perturbative QCD.

Our model will be completed as an event generator if combined with some hadronization models. Then it can be used for reliable estimation of the detector efficiency in experimental analysis. On the other hand the model will be useful for theoretical investigations, as the model is rigorously based on perturbative QCD. For example, it is easy to change the input parton distributions at Q_0^2 in our model. Thus we can study in detail how the change of the input distributions affects the final results. Also interesting is the study on charm distributions. However, the value of Q_{0c}^2, where the photon starts to resolve into charm quark pair, is not clear. Naively one expects $4m_c^2$, but kinematically it should be $4m_c^2/x_b$, where x_b is a Bjorken variable. The parton shower model presented in this work would give us a quantitative answer to the question whether this kinematical change modifies the parton distributions or not. Extensive studies on these points will be presented in a forthcoming paper.

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Figure Captions

Fig.1   The schematics of deep inelastic scattering of photon(a) and two quasi-real photons process(b) in e^+e^- scattering. Here γ denotes a quasi-real photon, while γ^* denotes the virtual one.

Fig.2   Parton distributions $xf(x, Q^2)/\alpha$. The solid and dashed curves are calculated by the parametrized form given in ref.[14] for LL approximation. The circles and diamonds show results of our shower model with a constant cutoff, which should be very small compared to the observed $(1-x)$ value. Here it is 10^{-4}. The evolution starts from $Q^2 = 0.25\text{GeV}^2$ by taking common initial distributions. (a) u-quark distributions. (b) u-quark and gluon distributions.

Fig.3   The structure function $F_2(x, Q^2)/\alpha$. The curves are calculated by the authors of ref. [12] The diamonds are results by the shower model with p_T^2, while the plus symbols represent that with the virtual mass squared. These distributions are normalized at $Q^2 = 4\text{GeV}^2$. The full circles are the experimental data[1]. $Q^2 = 73\text{GeV}^2$(a), 1000GeV^2(b).

Fig.4   Cross sections of jet production integrated over the transverse momentum in the two quasi-real photon process in e^+e^- scattering. The solid and dashed curves are calculated with the parton distributions of ref.[14]. Circles and diamonds are results of our parton generator. In the solid curve and circles the hard scattering contains only the photon-quark scattering, while in the dashed curve and diamonds it contains only photon-gluon scattering. The c.m.s energy is 58GeV.

Fig.5   Thrust distributions of the final partons in the two quasi-real photon process in e^+e^- scattering. This is calculated only by transverse momenta of the final partons. The histogram, circles and diamonds are results with $p_{Tmin}^2 = 20\text{GeV}^2$, 50GeV^2 and 100GeV^2, respectively. The c.m.s energy is 58GeV.
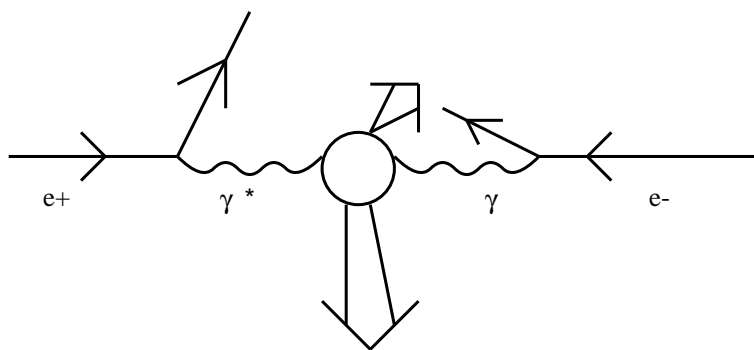
Fig.1(a)

Fig.1(b)
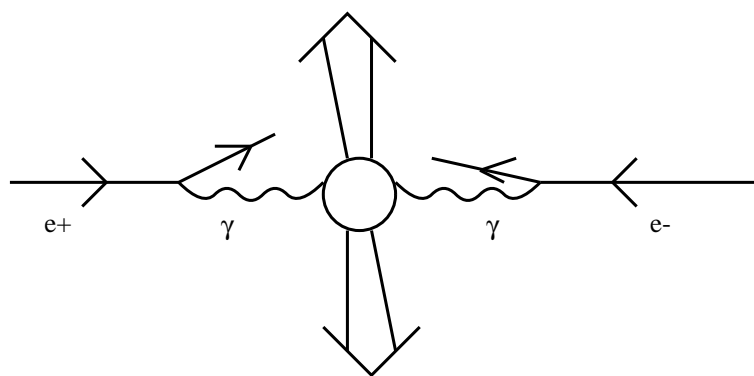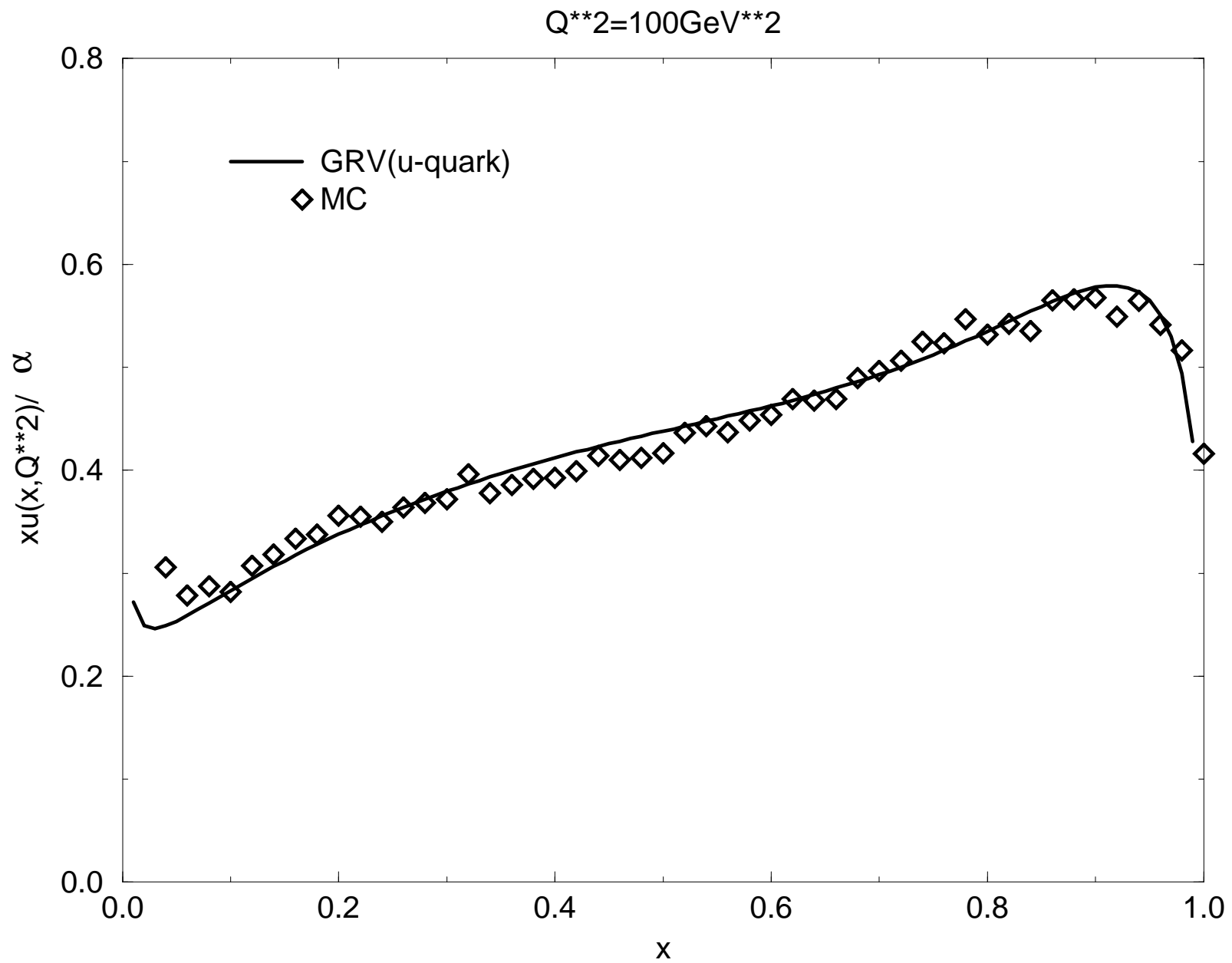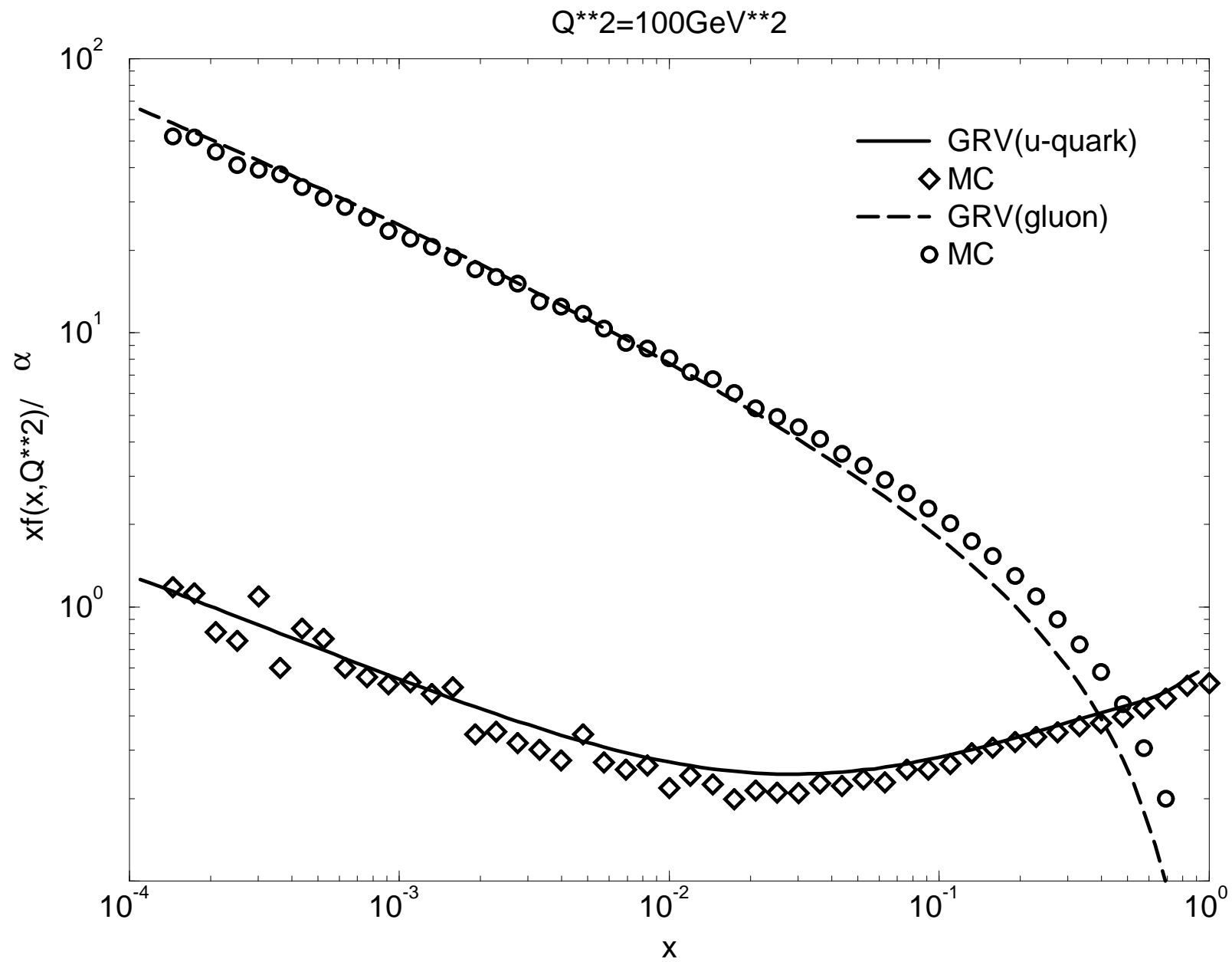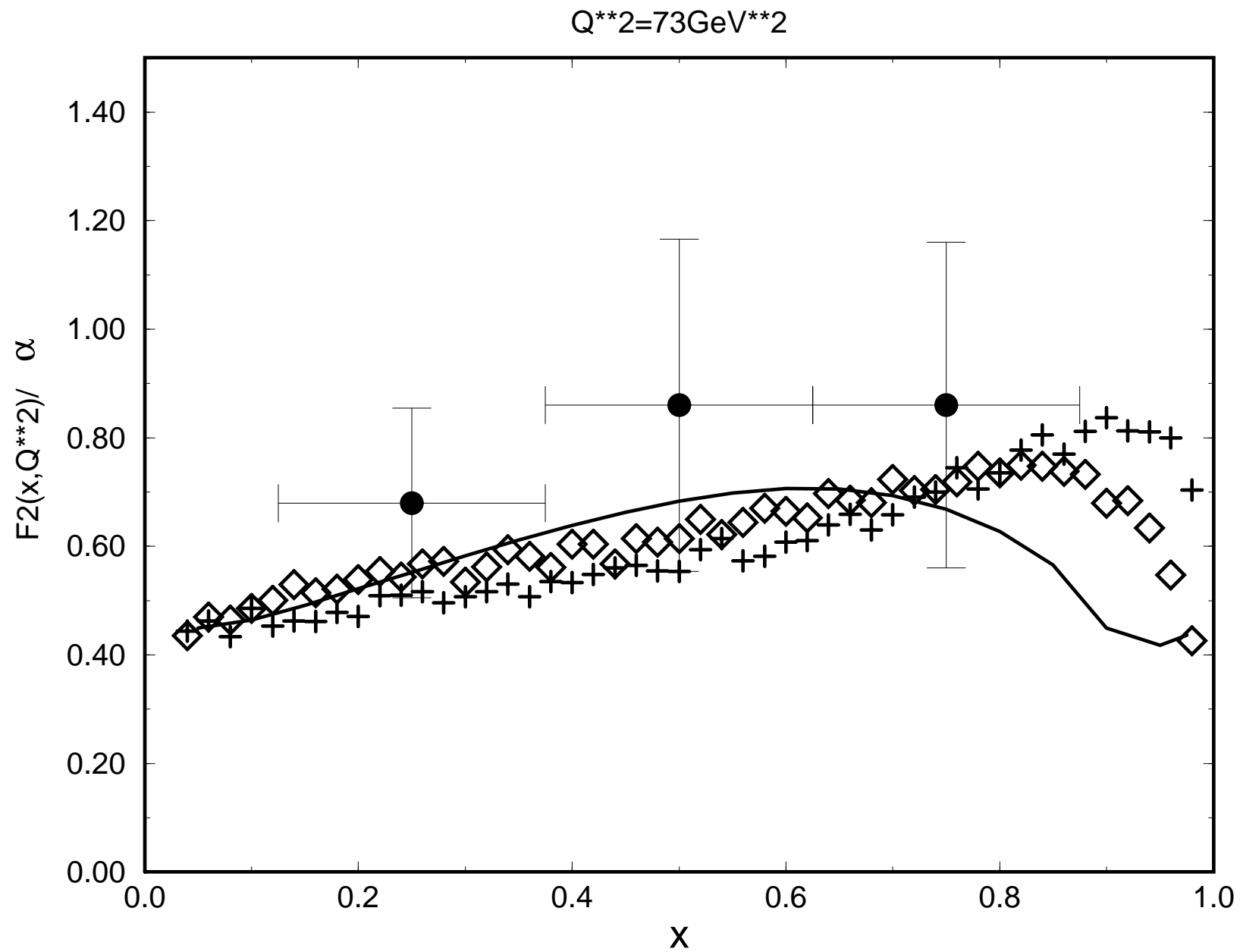
Fig.2(a)

Fig.2(b)

Q**2=73GeV**2

F2(x,Q**2)/ α

x

Fig.3(a)

Fig.3(b)

Fig.4

W=58GeV

Fig.5